COMPLEX ANALYSIS

Place and time: In M101 on Thursday, Jan 4, at 16:00–17:30
Organizers: Janne Heittokangas (University of Eastern Finland)
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On dominating sets and sampling measures for weighted Bergman spaces

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Abstract. Let $\mathbb{D}$ denote the unit disc in the complex plane. A function $\omega : \mathbb{D} \to [0, \infty)$, integrable over $\mathbb{D}$, is called a weight. For $0 < p < \infty$, the weighted Bergman space induced by a weight ω is the space of analytic functions f on $\mathbb{D}$ such that

$$\|f\|_{A^p_\omega}^p = \int_{\mathbb{D}} |f(z)|^p \omega(z)\, dA(z) < \infty.$$

For $\delta > 0$, a measurable set $G \subset \mathbb{D}$ is a δ-dominating set for $f \in A^p_\omega$ if

$$\int_G |f(z)|^p \omega(z)\, dA(z) \geq \delta \|f\|_{A^p_\omega}^p.$$

A measurable set $G \subset \mathbb{D}$ is a dominating set for the space A^p_ω if there exists $\delta > 0$ such that G is a δ-dominating set for all $f \in A^p_\omega$.

Inspired by the work of Luecking in the 1980s, we will discuss conditions for subsets of $\mathbb{D}$ to be dominating sets for functions in A^p_ω or for the space itself when the weight ω has a certain doubling property. We will also consider some properties of sampling measures of A^p_ω, that is, positive Borel measures μ on $\mathbb{D}$ satisfying

$$\int_{\mathbb{D}} |f(z)|^p \, d\mu(z) \asymp \|f\|_{A^p_\omega}^p, \quad f \in A^p_\omega,$$

as well as their connection to dominating sets.

Joint work with Jouni Rättyä.

Structural rigidity of generalised Volterra operators on Hardy spaces

SANTERI MIIHKINEN *(Åbo Akademi University)*, santeri.miihkinen@abo.fi

Abstract. Generalised Volterra operator

$$V_g f(z) = \int_0^z f(\zeta) g'(\zeta)\, d\zeta, \quad z \in \mathbb{D},$$

where f, g are analytic in the unit disc $\mathbb{D}$ of the complex plane, was introduced by Pommerenke in the 1970s in connection to exponentials of BMOA functions. Its properties have been widely studied by several authors in many analytic function spaces since the mid-1990s when its boundedness and compactness were characterized in Hardy and Bergman spaces by Aleman and Siskakis. In this talk, we look into its structural properties, ℓ^2-singularity in particular. An operator is ℓ^2-singular if its restriction to any subspace isomorphic to the sequence space ℓ^2

is not bounded below, i.e. it does not fix an isomorphic copy of ℓ^2. Our main result implies that generalised Volterra operators acting on Hardy spaces H^p are always ℓ^2-singular for $p \neq 2$.

Joint work with P.J. Nieminen, E. Saksman and H.-O. Tylli.

Vanishing Bergman kernels

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Abstract. Bergman kernels are central objects in complex analysis and related fields. In many instances, the methods (such as optimization, discretization etc.) involving Bergman kernels are only applicable if the kernel has no zeroes. Unfortunately, in a sense, this is hardly ever the case. On the other hand, not that many explicit formulas for vanishing Bergman kernels are known.

In this talk, we review the recent developments regarding this topic, and present a method for finding formulas for Bergman kernels with a prescribed number of zeroes.